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TOTAL UTILITY AND CONSUMERS' SURPLUS UNDER VARYING CONDITIONS OF THE DISTRIBUTION OF INCOME

IN discussions of the principle of consumers' surplus as developed by Marshall, the preliminary assumption has been that the diminishing utility curve might be represented by the demand curve and that marginal utility might be measured by price. This assumption is satisfactory for an individual, and may also be applied to a group of persons whose psychologic reactions to successive increments of all commodities are the same, provided that they have equal purchasing power. But in the case of a community whose members differ widely with respect to money income, the demand curve and the price no longer even approximately represent conditions of utility. The utility to the first purchaser, a millionaire, may be low, while the utility to the marginal purchaser, a worker in a sweat-shop, may be very high. The millionaire takes the first unit not because its utility to him is higher than to the worker but because the sacrifice involved in purchasing it is very much less.¹

So much is familiar. The following analysis is an attempt to adapt the concepts of total utility and consumers' surplus

¹ The above principle is recognized by Professor Marshall. "A pound's worth of satisfaction to an ordinary poor man is a much greater thing than a pound's worth of satisfaction for an ordinary rich man," (*Principles of Economics*, 6th ed., p. 130). He thinks, however, that this consideration may be disregarded in practice, because "it happens that by far the greater number of the events with which economics deals affect in about equal proportions all the different classes of society; so that if the money measures of the happiness caused by two events are equal, there is not in general any very great difference between the amounts of the happiness in the two cases." This is undoubtedly true with reference to the events which Professor Marshall had in mind. If, however, the event in question were a socialistic proposal looking to the redistribution of income or even a drastic minimum wage law, this principle, far from being negligible, becomes the determining factor for justifying the innovation.

to a community with unequal incomes — a community such as we find in the actual world.

First, assume a group made up of four persons (A, B, C, D) having the same money incomes, and further assume them to be alike in their utility reaction to successive increments of some commodity, say, oranges. As successive oranges are offered, each yields a diminishing utility. For brevity suppose that we denote a unit of psychic satisfaction derived from an orange by u . Then the satisfaction derived from the first orange may be assumed to be $5u$; from the second, $4u$; from the third, $3u$; from the fourth, $2u$; from the fifth, u . Satiety is reached with the sixth orange — the orange becomes a free good.

Now so long as incomes are equal, utilities may be measured by money payments. The measures of $5u$, $4u$, $3u$, then become 5 cents, 4 cents, 3 cents, and so on.

If a dealer puts four oranges on the market, each person will take one orange at 5 cents; if he puts eight oranges on the market, each will take two at 4 cents; if twelve, three at 3 cents; and so on. Under the hypothesis, as so far advanced, diminishing utility and demand are both represented by the double line, PX, in Figure 1.

Throughout this discussion, A, B, C, and D are assumed to be aggressive purchasers in the order of the alphabet. That is, if only one orange were put on the market, it will go to A at 5 cents. B, C, and D would also give 5 cents, and not one of the purchasers would give more than 5 cents. But A gets the orange. In like manner, if two oranges are offered, they go to A and B; if three, to A, B, and C.

Next, let us suppose the incomes of the members of our community to be redistributed so that for A, B, C, and D respectively they will stand in the ratio of 4: 3: 2: 1. Since the amount of money in the community and hence the total money income is not changed, their money incomes will now be (assuming that before the change each income was x) $1.6x$, $1.2x$, $0.8x$, and $0.4x$. Now, under these conditions, as was noted in the first paragraph, utility curves can no longer be constructed and compared on the basis of the money pay-

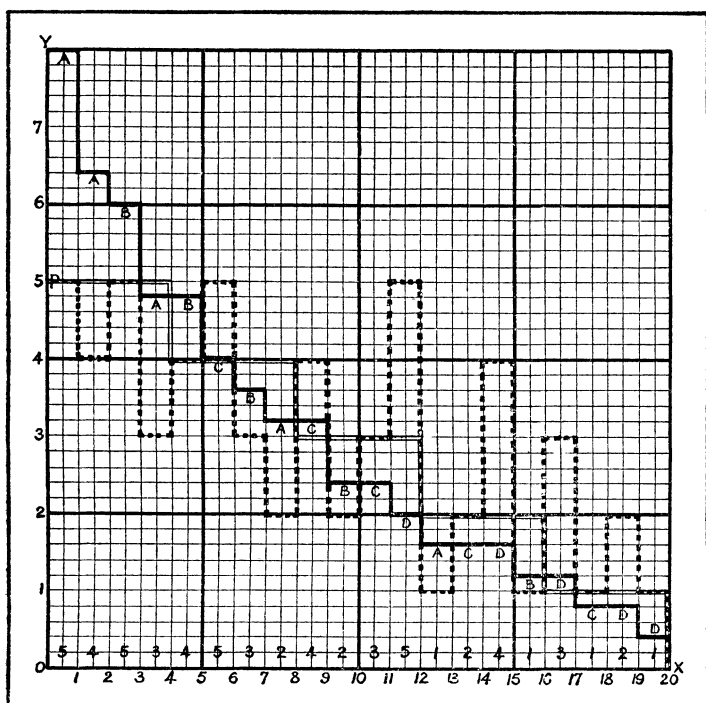


FIG. 1

Explanation:

— Demand or utility line, equal distribution of incomes.

--- Demand line, unequal distribution of incomes.

..... Utility line, unequal distribution of incomes.

Figures below line OX indicate supply.

Figures above line OX indicate corresponding utility of last unit taken, unequal distribution of incomes.

Letters A, B, C, D, indicate recipients of successive increments of supply, unequal distribution of incomes.

ments which would be given for successive increments of the commodity in question. A curve constructed on this basis will indeed be a *demand* curve, but not a *utility* curve. In order to make a comparison for utility between persons of different money incomes on the basis of their demand curves, some hypothesis is necessary for reducing money payment to utility.

The hypothesis which will be assumed in the following analysis is that the utility of any of the successive increments of a commodity to the purchaser may be measured by the fractional part of his entire money income which he is willing to sacrifice in order to obtain it. For example, if A's income is twice B's, and if B would pay x cents for a thing, then the fact that A would pay $2x$ cents for the same thing indicates that the utility of the thing is the same to both.

On this basis, a demand schedule may be made out for A, B, C, and D, separately and collectively, under their changed incomes. Since, when their incomes were equal, each would pay 5 cents for the first orange, now, when A's income is 1.6 times what it was before, he will pay 1.6×5 cents, or 8 cents. This method of computation may be applied to the first, second, third, fourth, and fifth orange for each of the buyers under their changed incomes. The result of the computation is shown in the following demand schedule.

TABLE I

At	A takes	B takes	C takes	D takes	Group takes
8	1	..	..	..	1
6.4	2	..	..	..	2
6	2	1	..	..	3
4.8	3	2	..	..	5
4	3	2	1	..	6
3.6	3	3	1	..	7
3.2	4	3	2	..	9
2.4	4	4	3	..	11
2	4	4	3	1	12
1.6	5	4	4	2	15
1.2	5	5	4	3	17
.8	5	5	5	4	19
.4	5	5	5	5	20
cents	oranges	oranges	oranges	oranges	oranges

The demand "curve" for the group (derived from this schedule) is shown by the full-line steps (YX) in Figure 1.

Now if by consumers' surplus we mean nothing more than the money which the members of a community would have paid in excess of what they actually do pay, for a given supply

of a commodity, offered in successive increments, rather than go without it, then from a study of the above schedule and Figure 1 it at once becomes apparent that in this sense consumers' surplus is in general greater under conditions of unequal than of equal distribution of incomes, until the supply reaches satiety, when the two results will be the same. The last part of this statement is obvious, *a priori*; for at satiety the sum actually paid is 0 while the sum which would have been spent is the same in each case.¹

The following table gives the consumers' surplus computed as above under different conditions of supply.

TABLE II

Oranges offered	Consumers' Surplus Equal distribution	Consumers' Surplus Unequal distribution
1	0	0
2	0	1.6
3	0	2.4
4	0	6.0
5	4	6.0
6	4	10.0
7	4	12.4
8	4	15.2
9	12	15.2
10	12	22.4
11	12	22.4
12	12	26.8
13	24	31.6
14	24	31.6
15	24	31.6
16	24	37.6
17	40	37.6
18	40	44.4
19	40	44.4
20	40	52.0
21 (Satiety)	60	60.0

¹ Under equal distribution A or B or C or D would each have spent 5 + 4 + 3 + 2 + 1 cents or 15 cents rather than go without the oranges. In all 60 cents. Under unequal distribution the amounts which would have been spent by each is shown by the following table.

$$\begin{array}{rcl}
 A & = & 1.6 \text{ (5 + 4 + 3 + 2 + 1)} \\
 B & = & 1.2 \text{ (" " " ")} \\
 C & = & .8 \text{ (" " " ")} \\
 D & = & .4 \text{ (" " " ")}
 \end{array}$$

$$A + B + C + D = 4.0 \text{ (" ")} = 60 \text{ cents.}$$

But such an analysis of consumers' surplus is without significance. Consumers' surplus, as a constituent of psychic income, is to be defined as "the difference between the sum which measures total utility and that which measures total exchange value."¹ But when persons have different incomes, sums of money expended or which might have been expended no longer measure either marginal or total utility. Before they can do so they must be reduced to terms of equal income. When A, in the supposed case of unequal income buys his first orange for 8 cents he is obtaining a utility of $5u$ and is making a money sacrifice (in terms of any other commodity he might have purchased) also of $5u$. In like manner the second orange at 6.4 cents represents a utility of $4u$ and a money sacrifice of $4u$. If only two oranges are offered, A gets them both at a money sacrifice equivalent to $4u$ each or $8u$ for both. Rather than go without them he would have made a money sacrifice equivalent to $9u$. He thus gains what may be called a "real" consumers' surplus of $1u$. Now if incomes had been equal and only two oranges had been offered, A and B would each have obtained an orange for 5 cents and each would have represented a utility of $5u$. Since by hypothesis neither would have paid more than 5 cents there would have been no consumers' surplus. It thus appears that "real" consumers' surplus, as above interpreted, may also be greater under inequality of income. It would be possible to determine the consumers' surplus in this sense for every increment of supply from two oranges to twenty-one. It may be worth while to note that in this sense also the consumers' surplus when satiety is reached will be the same for any distribution of income. But except for this comparatively unimportant deduction the investigation would be of little worth. At any rate it has no significance in comparing the well-being of a community having unequal distribution of wealth with one having equal distribution. For the consumers' surplus of those who purchase goods because of larger means rather than because of greater marginal utility is to some extent offset by a consumers' deficit

¹ Taussig, *Principles of Economics*, vol. i, p. 128.

(so to speak) on the part of those who are unable to purchase at all, because of insufficient means, even tho the goods have high utility.

To compare the well-being of the two communities the method of consumers' surplus is therefore inapplicable. Yet the method outlined above may be employed to compare them with respect to total utility as successive instalments of goods are offered; and this comparison will be significant. Using the same assumptions as in the case of "real" consumers' surplus we derive the following table.

TABLE III

Oranges offered	Total Utility Equal distribution					Total Utility Unequal distribution				
	A	B	C	D	Group	A	B	C	D	Group
1	5	..	..	..	5	5	..	..	..	5
2	5	5	..	..	10	9	..	..	..	9
3	5	5	5	..	15	9	5	..	..	14
5	9	5	5	5	24	12	9	..	..	21
6	9	9	5	5	28	12	9	5	..	26
7	9	9	9	5	32	12	12	5	..	29
9	12	9	9	9	39	14	12	9	..	35
11	12	12	12	9	45	14	14	12	..	40
12	12	12	12	12	48	14	14	12	5	45
15	14	14	14	12	54	15	14	14	9	52
17	15	14	14	14	57	15	15	14	12	56
19	15	15	15	14	59	15	15	15	14	59
20	15	15	15	15	60	15	15	15	15	60

Comparing the total utilities for the group it appears that for every condition of supply of over one orange or less than nineteen the total utility enjoyed by the group is less under conditions of unequal distribution of incomes than under equal distribution. The comparison of total utilities under the two conditions of distribution may also be made directly from the diagrams, Figure 1, Figure 2, the total utility for equal distribution being the area below the double line PX,

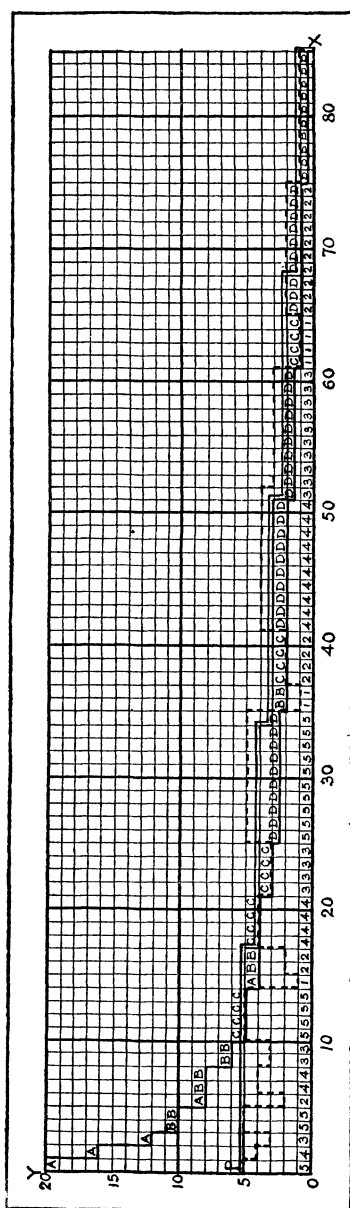


FIG. 2

Explanation:

— Demand or utility line, equal distribution of incomes.

— Demand line, unequal distribution of incomes.

..... Utility line, unequal distribution of incomes.

Figures below line OX indicate supply.

Figures above line OY indicate corresponding utility of last unit taken, unequal distribution of incomes.

Letters A, B, C, D, indicate recipients of successive increments of supply, unequal distribution of incomes.

and extending to the ordinate corresponding with the supply; and the total utility for unequal distribution being the area, to the same ordinate, below the dotted line, PX.

The utility "curves" corresponding with Table III are shown in Figure I, the double line, PX, indicating utility when incomes are equal; the dotted line, PX, when incomes are unequal. The ups and downs of the latter line clearly show that under conditions of unequal distribution of incomes the taking of successive "doses" of a commodity no longer indicate a steadily diminishing utility to the purchaser. The twelfth orange, for example, has for its purchaser a much higher utility than the tenth has for its purchaser.

This method may also be employed to compare the well-being of a community under equal distribution of income with that of the same community divided into non-competing groups having different incomes for the members of different groups. Assume a society made up of four non-competing groups, A, B, C, and D, with one person in group A, two persons in group B, four in group C, and ten in group D—seventeen in all, and assume individual incomes for the members of the groups to be in the ratio A: B: C: D = 8: 4: 2: 1. Assume the utility of oranges to members of all the groups to be as in the first problem. Then under conditions of equal income seventeen oranges would be taken at 5 cents, thirty-four at 4 cents, and so on, as shown by the double line, PX, in Figure 2. If under equal distribution each person has an income of x cents, then when the incomes are redistributed in the assumed proportions, each person in group A will have $4x$ cents; in group B, $2x$ cents; in group C, x cents; and in group D, $\frac{1}{2}x$ cents. From this we may construct the demand schedule on p. 316.

The last column of this demand schedule is represented by the full line steps YX, in Figure 2, the letters A, B, C, D, indicating the recipients of successive oranges. If, now, we multiply each money payment of A by $\frac{1}{4}$, of B by $\frac{1}{2}$, of C by 1, and of D by 2, we can construct the utility "curve" shown by the dotted line, PX. As in the former case it will be noted that under conditions of unequal incomes marginal

TABLE IV

At	Each A takes	Each B takes	All B's take	Each C takes	All C's take	Each D takes	All D's take	Whole Community takes
20	1	..	..	..	..	..	..	1
16	2	..	..	..	..	..	..	2
12	3	..	..	..	..	..	..	3
10	3	1	2	..	..	..	..	5
8	4	2	4	..	..	..	..	8
6	4	3	6	..	..	..	..	10
5	4	3	6	1	4	..	..	14
4	5	4	8	2	8	..	..	21
3	5	4	8	3	12	..	..	25
2½	5	4	8	3	12	1	10	35
2	5	5	10	4	16	2	20	51
1½	5	5	10	4	16	3	30	61
1	5	5	10	5	20	4	40	75
½	5	5	10	5	20	5	50	85
cts.	Oranges							

utility may increase as well as diminish with successive increments of supply. The total utility is shown by the area below the double line, PX, when incomes are equal; or below the dotted line, PX, when incomes are unequal. Computations of these areas yield the schedule on p. 317.

As before it will be noticed that between one orange and satiety the total utility is in general greater for equal than for unequal distribution of incomes.

We have assumed that the same kind of goods (oranges) are consumed whether incomes are equal or unequal. This would probably not be true in life. When incomes are unequal, altho A would give 20 cents for his first orange rather than go without it, he would probably prefer to give 20 cents for something having a higher utility than an orange (say a grape fruit) which was before inaccessible but now becomes accessible because of his greater purchasing power. In like manner each D, if the price of an orange were beyond his purchasing power, would probably take something of less utility, but

TABLE V

Oranges offered	Equal distribution	Total Utility Unequal distribution
1	5	5
2	10	9
3	15	12
5	25	22
8	40	32
10	50	38
14	70	58
21	101	79
25	117	91
35	156	141
51	204	191
61	224	221
75	245	245
85	255	255

cheaper (say, an apple). Inequality of incomes, therefore, leads to the production of types of goods, covering a greater range of expensiveness than equality of incomes.

Again, the objection may be made to the above method of analysis that as a person spent his money its marginal utility would increase and that therefore it would be inaccurate to multiply the utility of successive oranges by a *constant* derived from the marginal utility of money, to find what a person would spend. This is a matter of psychology. The objection would be valid if we consider each person as starting with an unreplaceable fund which he is steadily depleting. If, however, we consider the money as an income received in instalments contemporaneously with the spending the objection would be covered. In the case of incomes received by the week or month the matter is largely one of individual psychology. To some persons the utility of a dollar seems about the same during the entire interval between successive instalments of income. To others undoubtedly a dollar seems much more precious towards the end of the month than immediately after pay day.

Considerations such as these must undoubtedly preclude the hope of attaining mathematical precision in applying our analysis to real life. Indeed, such precision cannot be expected

from any analysis of so ideal and personal a thing as psychic income. The mathematical method, however, is often valuable in helping us to perceive and grasp a truth even where its results are not to be taken too literally; and no more is hoped for it in the present instance.

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